**Group 2**

**MLDS 440**

**Homework 6**

**Due 11/6/2023**

1. Given that our goal is to maximize revenue with a single price, we can use information about the willingness to pay of the customers to select the best price. In particular, we know that the distribution of how customers value the product is uniform between $0 and $1000 which gives an approximation of how many customers will buy the product at any given price point. For example, if we decide to charge $300, we can expect 70 people to buy the product yielding an expected revenue of $21,000. Following this logic, **we would charge $500 because this price will maximize the expected revenue at $25,000.**
2. Given a capacity of 15, **we would charge $850 to maximize the expected revenue.** The idea is that we want to choose the highest price point for which we can expect to sell to exactly 15 customers. Given the nature of the uniform distribution between $0 and $1000, it follows that we would expect 15 people to buy the product at $850. There is clearly no point charging a price below this to increase demand because of the capacity constraint. It also follows that the increasing the price would result in a drop in expected revenue since the loss in units sold outweighs the gain from the higher price.
3. Given the new potential for price segmentation, **we would charge $250 for the first 50 customers and $500 for the second segment.** The strategy in this context is to maximize revenue for each segment separately. For the first segment, we are dealing with a uniform distribution between $0 and $500 for 50 people. Conceptually similar to the problem in *part a*, the midpoint allows for reaching the maximum expected revenue: hence, $250. For the second segment, notice that the expected revenue for a price of $500 is $25,000. In fact, the tradeoffs present in this segment are identical to what we encountered in *part a*. As a result, increasing the price above $500 would result in a drop in expected revenue leading to $500 as the optimal price for the second segment. We did better here than the results in the first tab because we have additional information about the market. More specifically, the ability to charge varying prices for different sets of customers allows us to capitalize on information that is not possible by only charging a single price.
4. Given the capacity constraint with price segmentation, **we would charge $850 for both segments.** More specifically, the price of the first segment can be anything as long as it is high enough that no customers from the first segment will purchase the product. In other words, any price above $500 would work for the first segment. Since the capacity is 15, we have no reason to sell to customers in this segment and want to sell exclusively sell to customers in the second segment. For the second segment, we want to sell at the highest price point for which 15 customers are expected to purchase the product for the same reasons mentioned in *part b*: thus, a price of $850. We did pretty much identical here in terms of revenue relative to the second tab because we are actually solving the exact same problem.
5. Even if the capacity were to be 40, **only the pricing strategy for the second segment would change**. **We would still charge a price above $500 for the first segment** because we want to sell exclusively to the second segment in order to maximize revenue. For the second segment, the only difference is that now we want to select the highest price for which we can expect to sell to 40 people. Given a uniform distribution around $500 to $1000 with 50 customers, it follows that a price of $600 satisfies this condition. **Hence, we would charge a price of $600 for the second segment.**
6. **The critical fractile ratio is calculated using the formula:**

Where is the full-fare price, is the economy fare price, and represents the inverse of the cumulative distribution function of the demand.

Given that and , we compute the critical fractile ratio as:

By comparing the critical fractile with the empirical distribution, we determine the protection level ***Q\**** as the full-fare demand ***Q*** at the smallest cumulative probability greater than or equal to the critical fractile ratio.

Protection level: **46**

The booking limit for economy seats (BL) is calculated as the total number of coach seats minus the protection level.

Booking Limit (BL): **184**

1. **Expectation:** We expect the protection level ***Q\**** to be **Higher.**

**Justification:** The opportunity to sell unsold seats at the last minute at a reduced rate reduces the opportunity cost of reserving a seat for a full-fare customer. This can potentially increase the protection level since the risk of a seat remaining unsold is mitigated by the possibility of last-minute sales.

* 1. **How many slots should WZMU sell in advance?**

Difference in Revenue: The difference between selling a slot last-minute to political candidates and selling it in advance is:

$10,000 (last-minute) - $4,000 (in advance) = $6,000.

This $6,000 is the potential extra revenue that WZMU can earn from selling a slot last-minute instead of in advance.

The ratio of the guaranteed revenue from selling a slot in advance to the potential extra revenue from selling it last-minute is:

$6,000/$10,000 = 0.6.

This implies that if there's a 60% chance or greater of selling a slot last-minute, WZMU should consider not selling it in advance to potentially earn the extra revenue.

Using the Cumulative Probability Table to find where the cumulative probability is greater than 0.6

From the table provided, the cumulative probability closest to 0.667 is 0.7 for d=13.

This means there's a 60% chance that last-minute demand will be 13 slots or fewer.

Conclusion: Given the potential extra revenue, WZMU should protect (or not sell in advance) 13 slots, hoping to sell them last-minute to political candidates. This means they should sell 25 - 13 = 12 slots in advance.

* 1. **If a slot not sold in advance or last-minute is worth $2,500 for promotional messages:**

New Difference in Revenue:

$10,000 (last-minute) - $2,500 (promotional) = $7,500.

This $7,500 is the potential extra revenue from selling a slot last-minute instead of using it for a promotional message.

$7,500/$10,000 = 0.75

This means if there's a 75% chance or greater of selling a slot last-minute, WZMU should consider not using it for a promotional message to potentially earn the extra revenue.

Look for the highest demand (d) for which the cumulative probability is less than or equal to 0.75. From the table, the cumulative probability closest to (but not exceeding) 0.75 is 0.8 for d=15.

Conclusion: Given this potential extra revenue, WZMU should protect (or not use for promotional messages) 15 slots, hoping to sell them last-minute. This means they should sell 25 - 15 = 10 slots in advance and keep the remaining 11 slots either for last-minute sales or for promotional messages.

So, in conclusion:

a) WZMU should sell 12 slots in advance.

b) WZMU should sell 10 slots in advance, given the potential value of unsold slots.